Scalable Algorithms for Distribution Search

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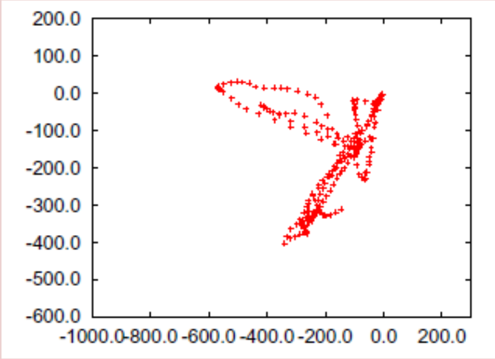
Introduction

Main intuition and motivation

Example: Motion capture

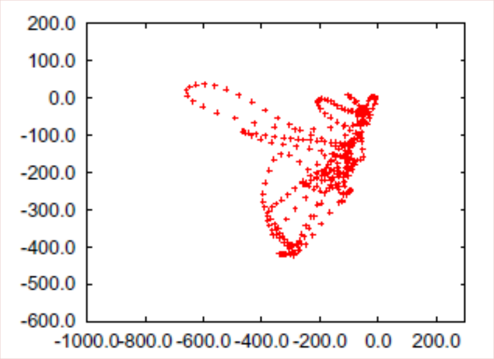
The scatter plots of foot kinetic energy values

#1 and #2 are similar and dissimilar distributions, respectively.



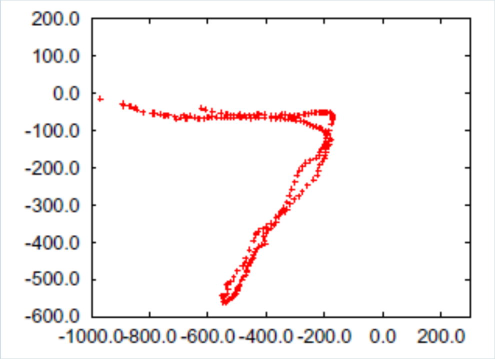
Query : Q

(running)



Distribution #1

(running)



Distribution #2

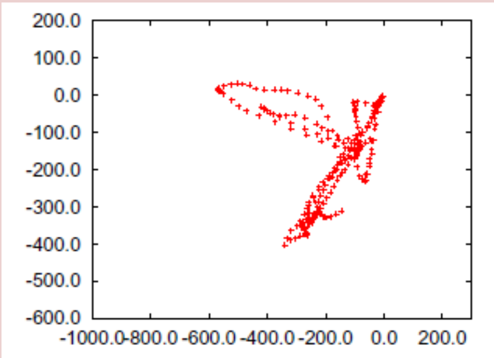
(squats)

- Our approach can identify Q and #1 as similar distributions

Problem definition

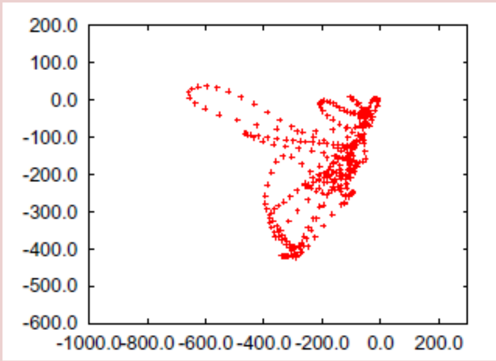
- Problem (Distribution search):

  "Given *n* distributions and query *Q*,
  Find similar distributions from the data set"



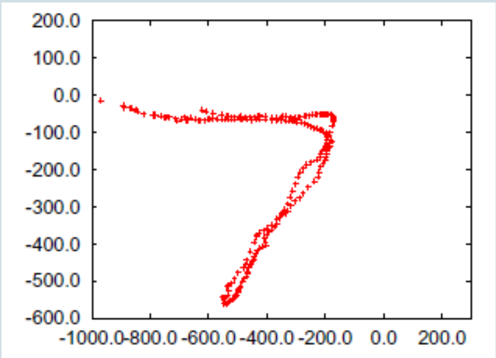
Query : Q

(running)



Distribution #1

(running)



Distribution #2

(squats)

Applications

- Distribution search application domains
  - Multimedia
  - Medical data analysis
  - Web service
  - E-commerce

Applications

- Multimedia

  Example: Motion capture datasets

  - Every motion can be represented as a cloud of hundreds of frames
  - For this collection of clouds, we can find similar motions without using annotations or other meta-data

Applications

- Web service

  Example: On demand TV

  - Discovering clusters and outliers in such data would help in tasks such as service design and content targeting

  (which groups or communities of users are associated with each other?)

Outline

- Introduction
- Background
- D-Search
- Time-series distribution mining
- Experiments
- Conclusions

Background

- Kullback-Leibler divergence

  Measures the natural distance difference
  from one probability distribution P
  to another arbitrary probability distribution Q.

$$d_{KL}(P,Q) = \int p_x \cdot \log\left(\frac{p_x}{q_x}\right) dx$$

* One undesirable property: $d_{KL}(P,Q) \neq d_{KL}(Q,P)$

- <u>Symmetric KL-divergence</u>

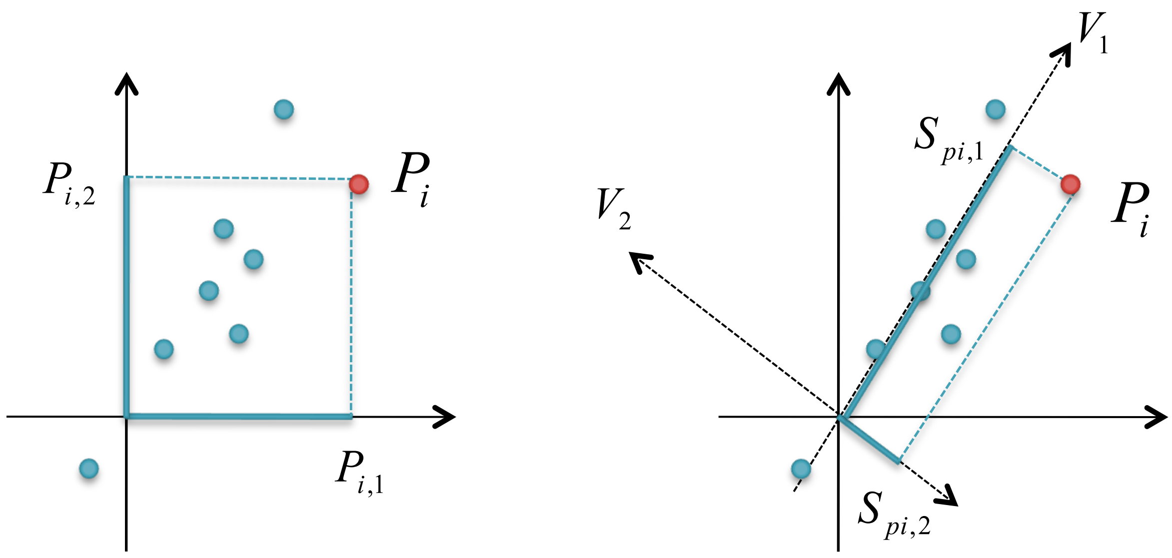
$$d_{SKL}(P,Q) = \int p_x \cdot \log\left(\frac{p_x}{q_x}\right) dx + \int q_x \cdot \log\left(\frac{q_x}{p_x}\right) dx = \int (p_x - q_x) \cdot \log\left(\frac{p_x}{q_x}\right) dx$$

Background

- Singular value decomposition (SVD)

  Every matrix $P \in \mathbf{R}^{m \times n}$ can be decomposed into

$$P = U \Sigma V^T$$



- The transformed data is given as:

$$S_p = U \Sigma$$

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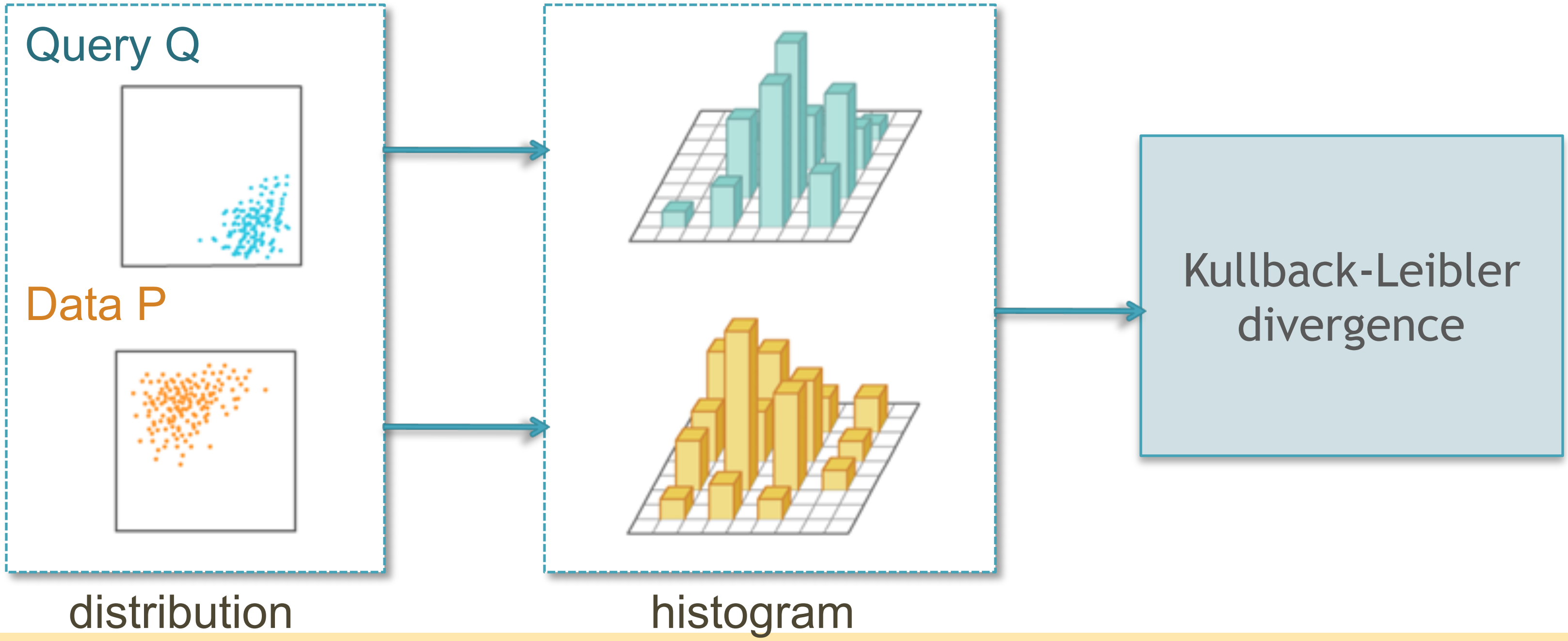
Proposed method

Naïve approach

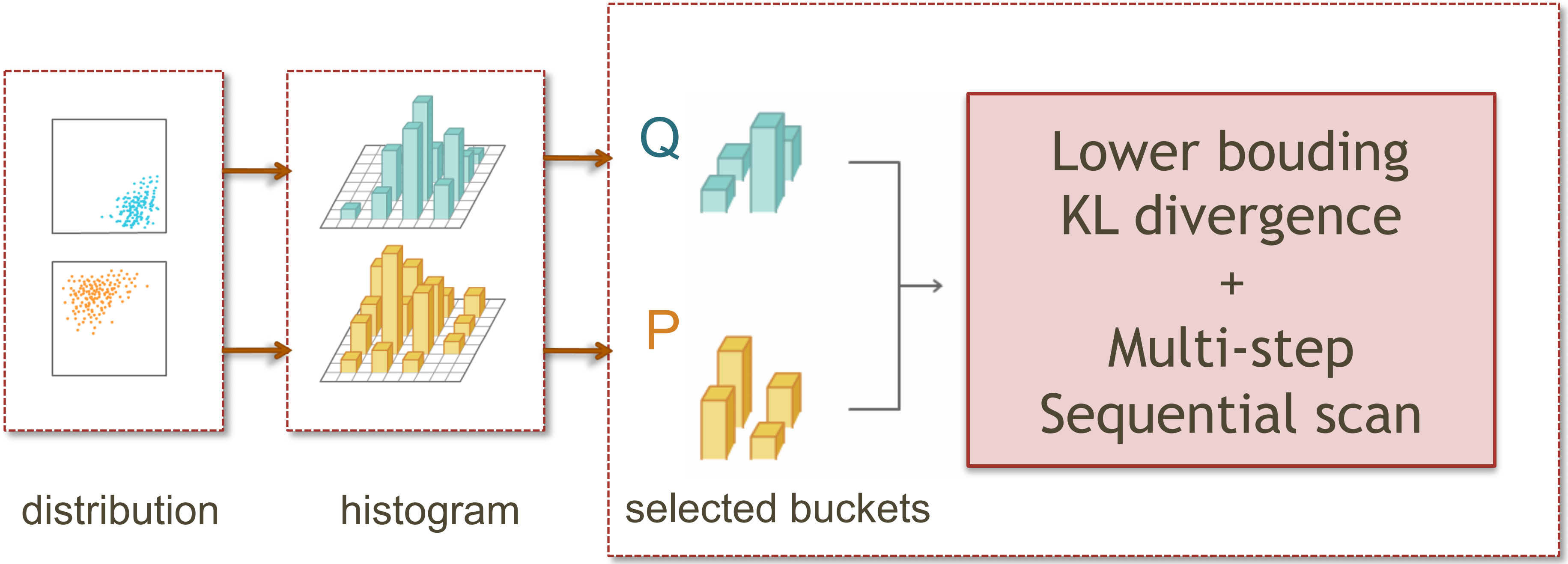
- Create histogram for each distribution of data
- Compute the KL divergence directly from histograms p_i and q_i
- Use any data mining method (k-nearest neighbor search)



Proposed method

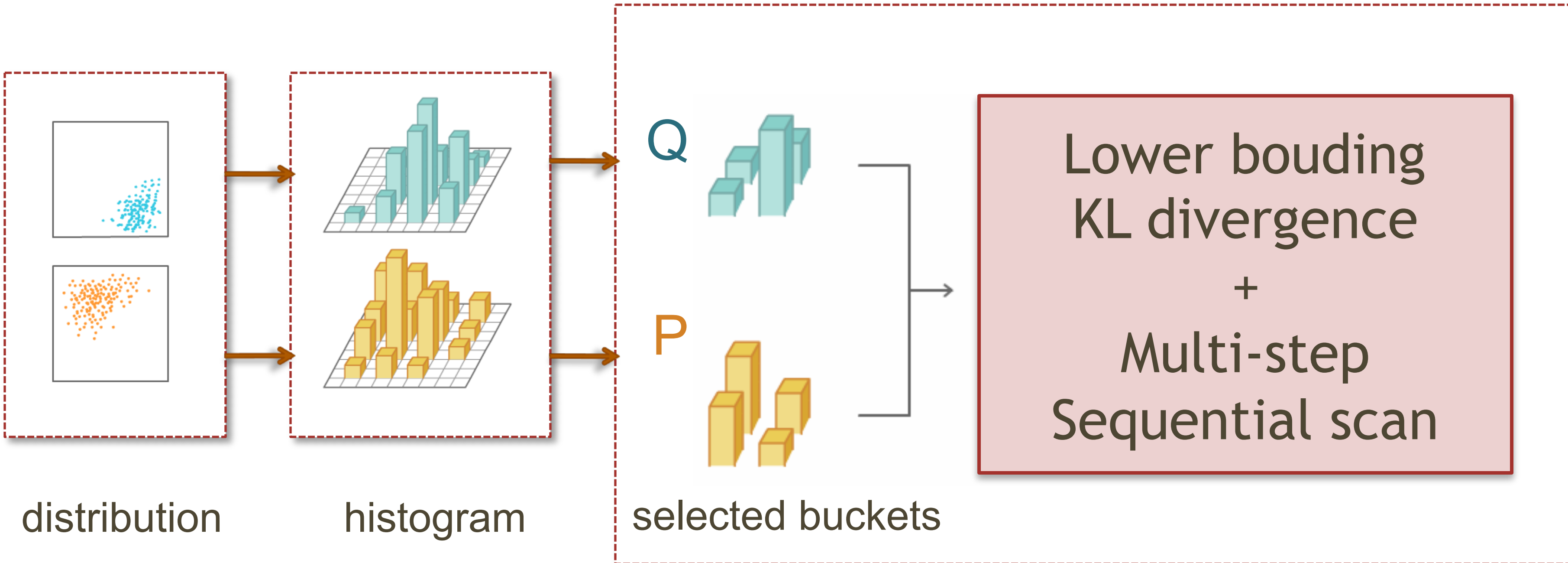
D-Search

- Compress histogram P and Q
- Compute the lower bounding KL divergence
- Prune the search candidates (Multi-step sequential scan)



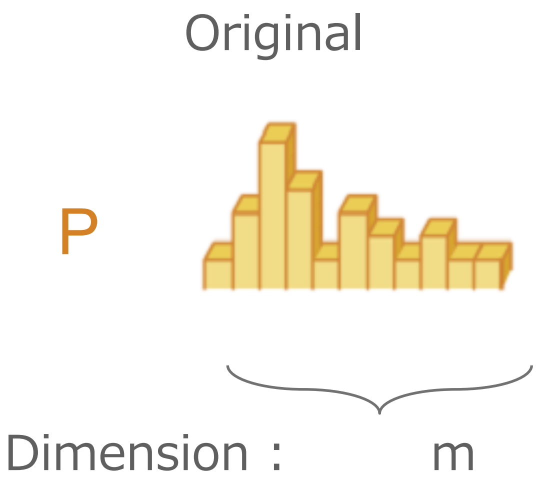
Proposed method

- **D-Search**
  - Compress histogram P and Q
  - Compute the lower bounding KL divergence
  - Prune the search candidates (Multi-step sequential scan)



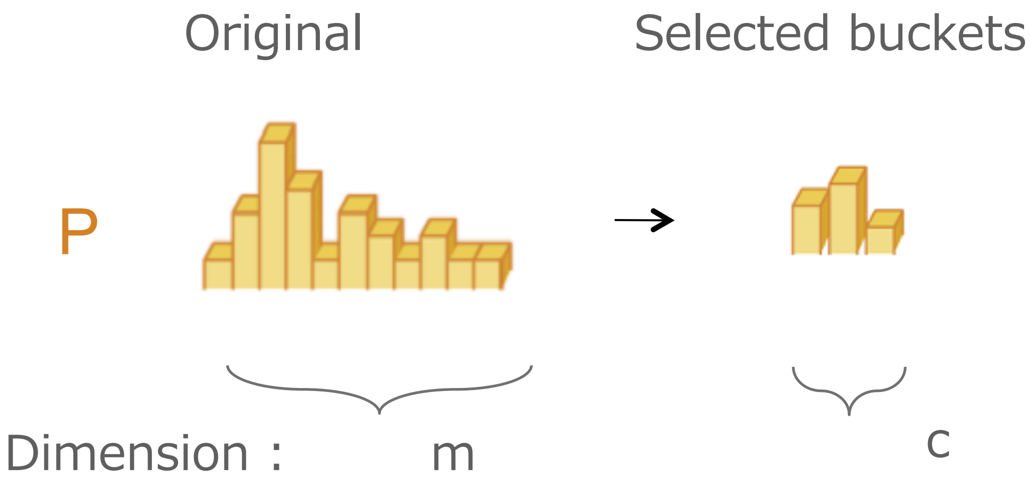
D-Search

- **Lower bounding KL divergence**
  - Create histogram for each distribution



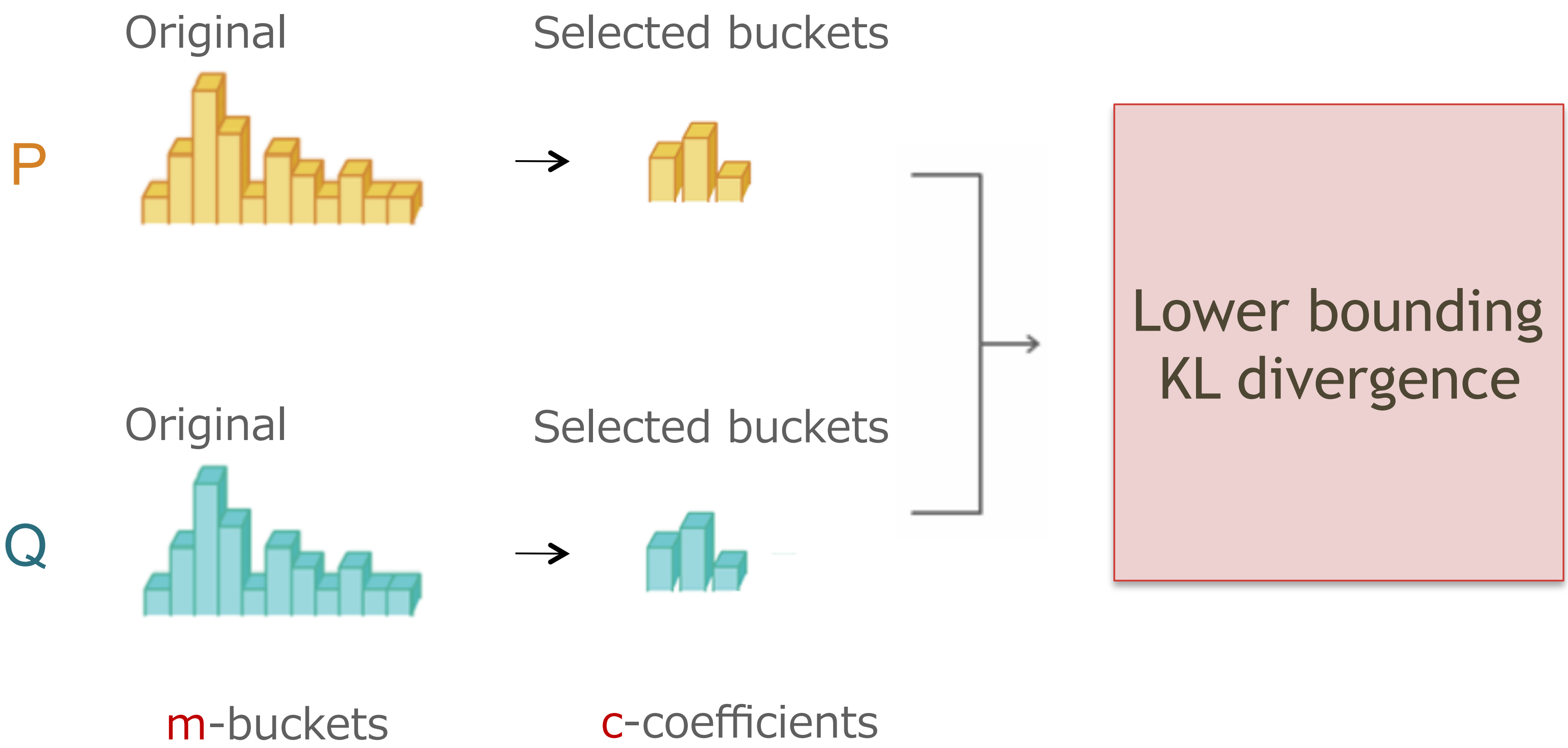
D-Search

- Lower bounding KL divergence
  - Create histogram for each distribution
  - Select the top c most populated buckets



D-Search

- Lower bounding KL divergence
  - Compute the KL divergence from the selected buckets



D-Search

- Lower bounding KL divergence
  - Compute the KL divergence from the selected buckets

$$d_c(P,Q) = \sum_{i \in I_{pq}} (p_i - q_i) \cdot \log\left(\frac{p_i}{q_i}\right)$$

$i \in I_{pq}$: Positions of the top c values

D-Search

Lower bounding KL divergence

- Compute the KL divergence from the selected buckets

$$d_c(P,Q) = \sum_{i \in I_{pq}} (p_i - q_i) \cdot \log\left(\frac{p_i}{q_i}\right)$$

$i \in I_{pq}$: Positions of the top c values

Lemma 1

$$d_{SKL}(P,Q) \geq d_c(P,Q)$$

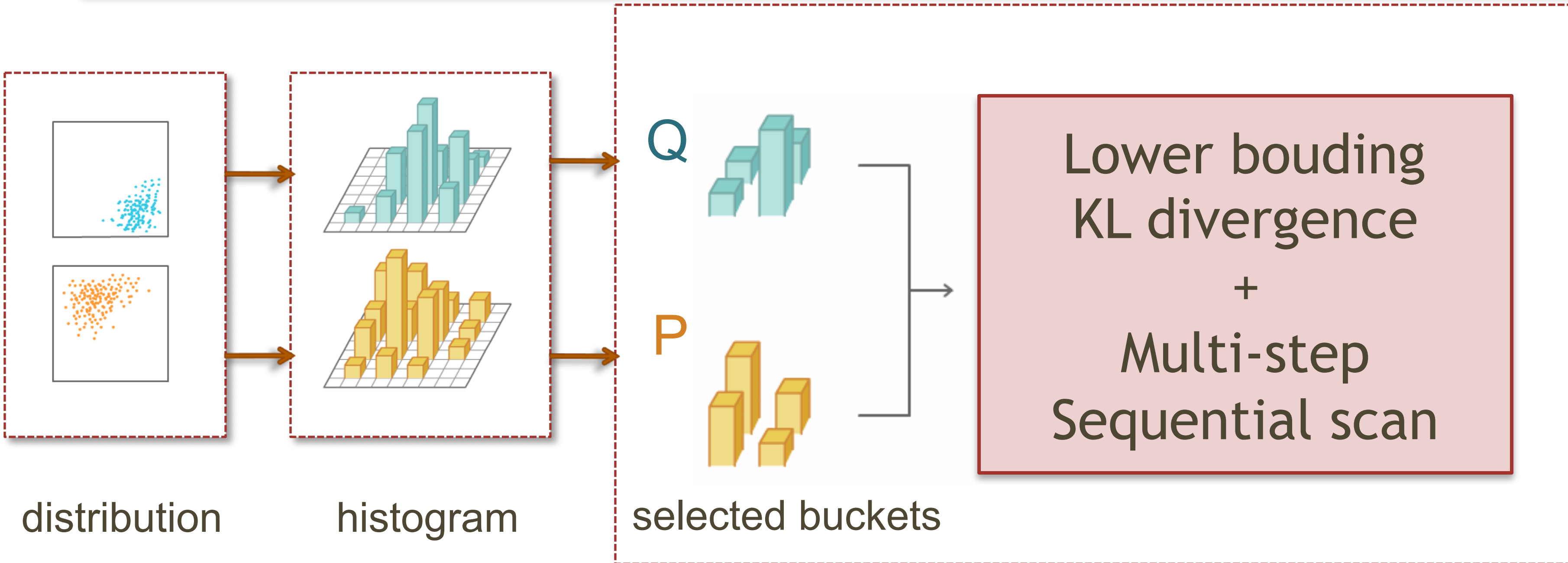
For any distributions, lower bounding KL divergence can be computed

$$\because \forall i, (p_i - q_i)(\log p_i - \log q_i) \geq 0$$

Proposed method

D-Search

- Compress histogram P and Q
- Compute the lower bounding KL divergence
- Prune the search candidates (Multi-step sequential scan)



D-Search

- Multi-Step Sequential Scan
  - KNN-search approach based on the lower bounding distance

    - Prune a significant number of search candidates
    - Lead to a direct reduction in the search cost

  - Guarantee no false dismissals

    (i.e., guarantee the exactness of search results)

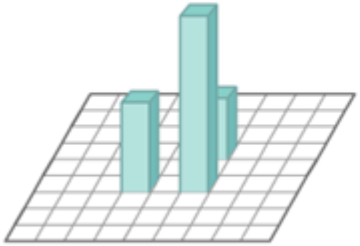
D-Search

- For the first step,
  - Compute the lower bounding distance from the coarsest version of P ((a) c=4)
  - If the distance is greater than D_{cb} (the current k-th nearest neighbor distance), we can prune P

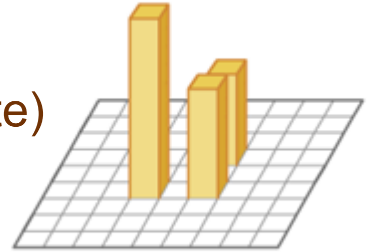
Step 1

(a) c = 4 (b) c = 8 (c) Original data

Q (query)

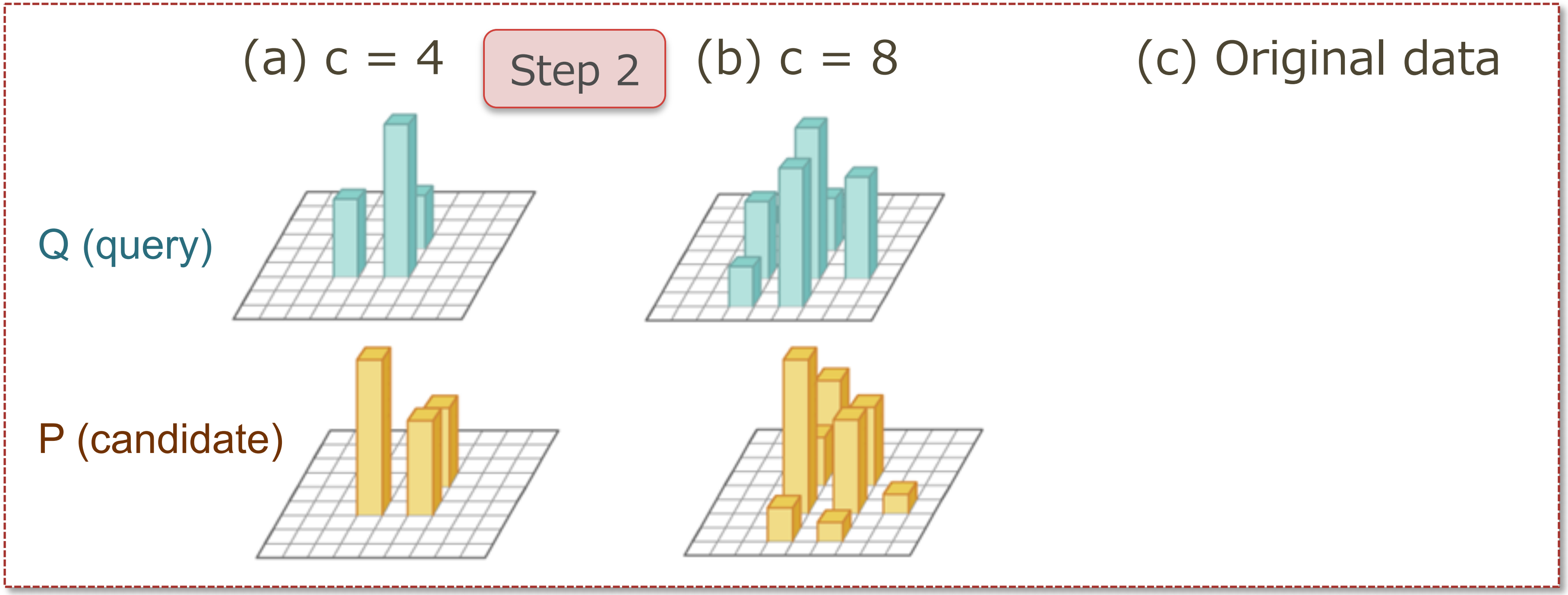


P (candidate)



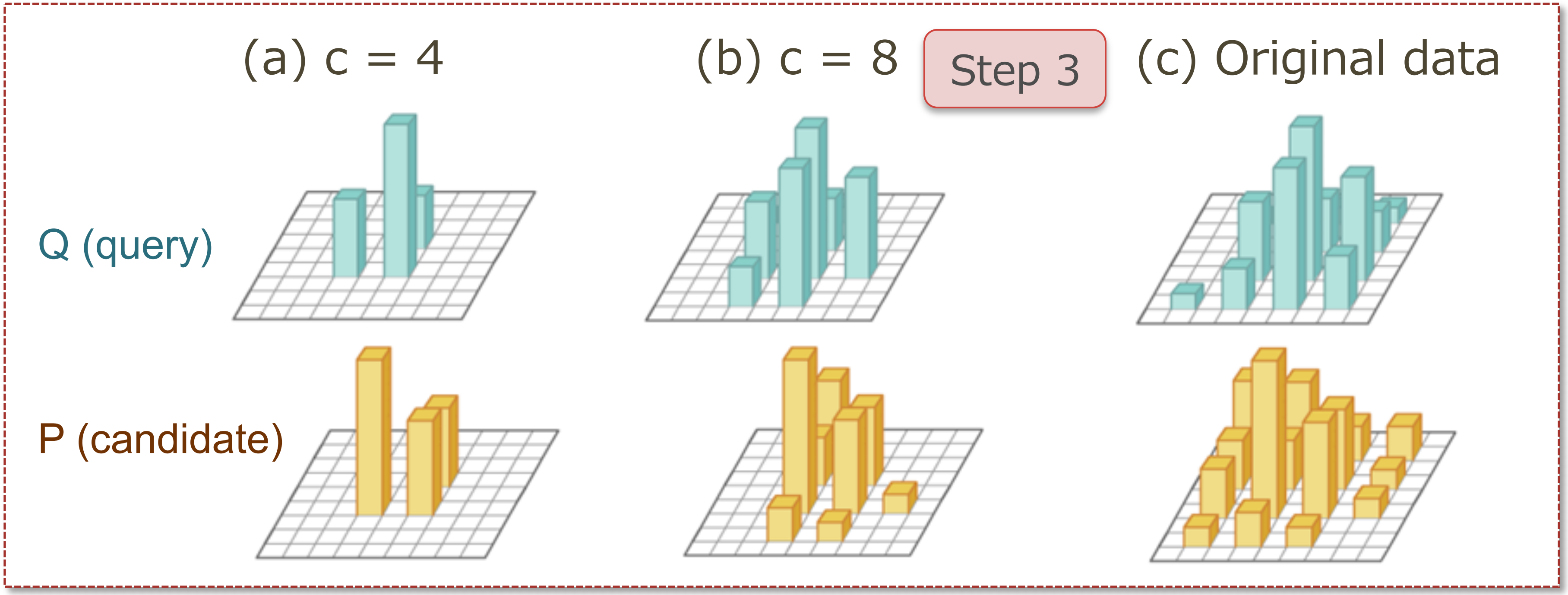
D-Search

- Otherwise,
  for the second step,
  - Compute the lower bounding distance from the more accurate version of P ((b) c=8)



D-Search

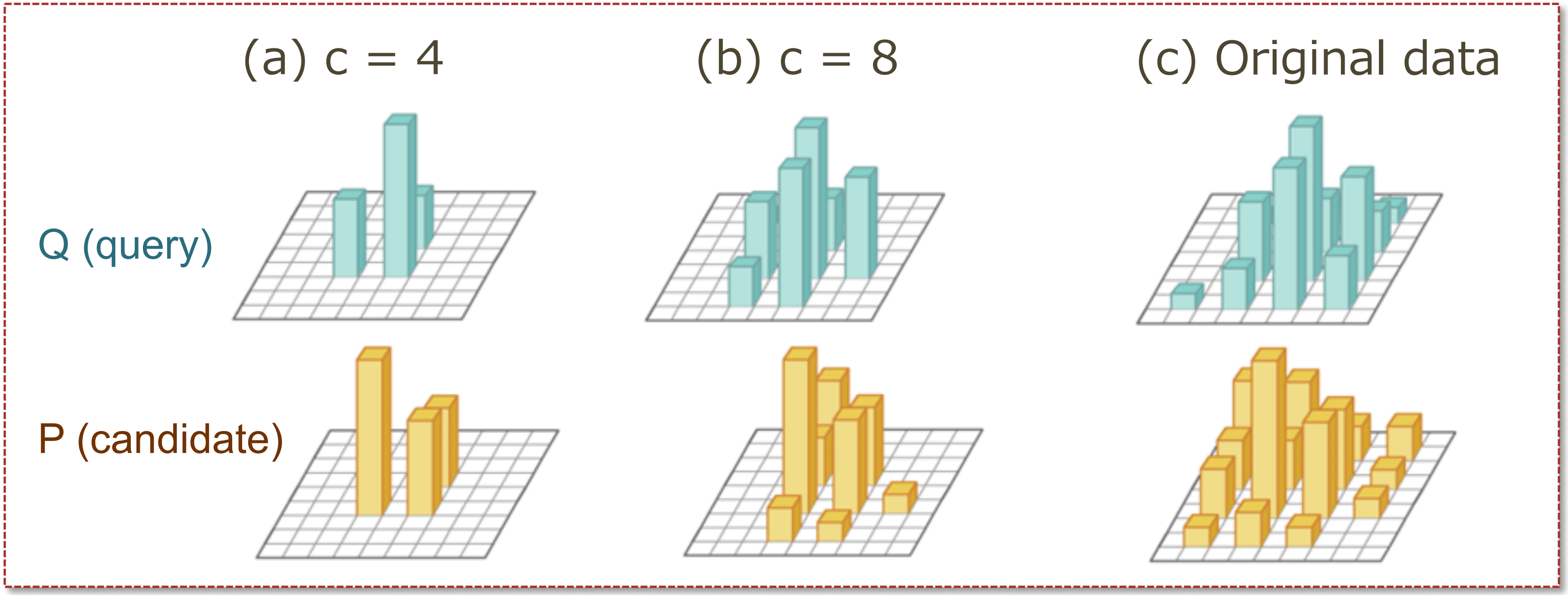
- If the lower bounding distance does not exceed D_{cb} for the third step,
  - Compute the exact distance of P ((c) Original data)



D-Search

- For the final step,
  If the exact distance does not exceed D_{cb}
  - Update the answer candidate and D_{cb}

- Repeat this procedure for every distribution



D-Search

- ## Enhanced D-Search

  More efficient solution without a theoretical guarantee

D-Search

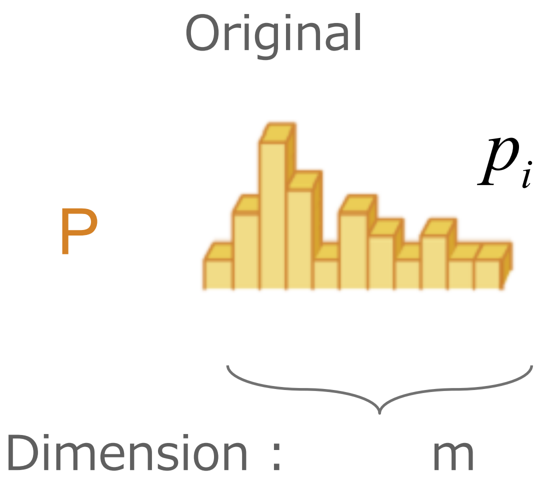
Enhanced D-Search

More efficient solution without a theoretical guarantee

- Compute the SVD coefficients of histogram P and Q
- Approximate the KL divergence

Enhanced D-Search

- Approximate KL divergence
  - Create histogram for each distribution



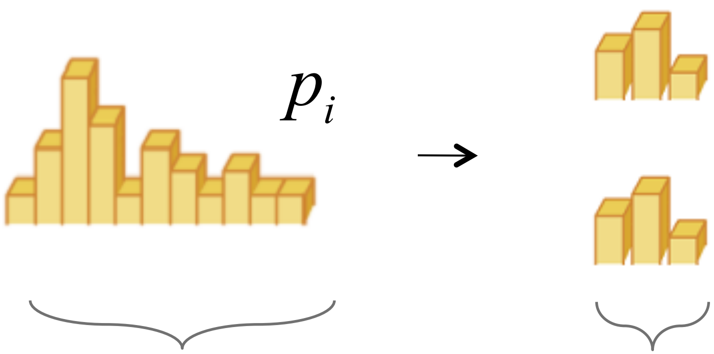
Enhanced D-Search

Approximate KL divergence

- Create histogram for each distribution
- Represent each histogram p_i and $\log(p_i)$ as sp_i and $s\hat{p}_i$ using SVDs
- Reduce the number of SVDs by selecting top *c*

Original SVD coeffs

P



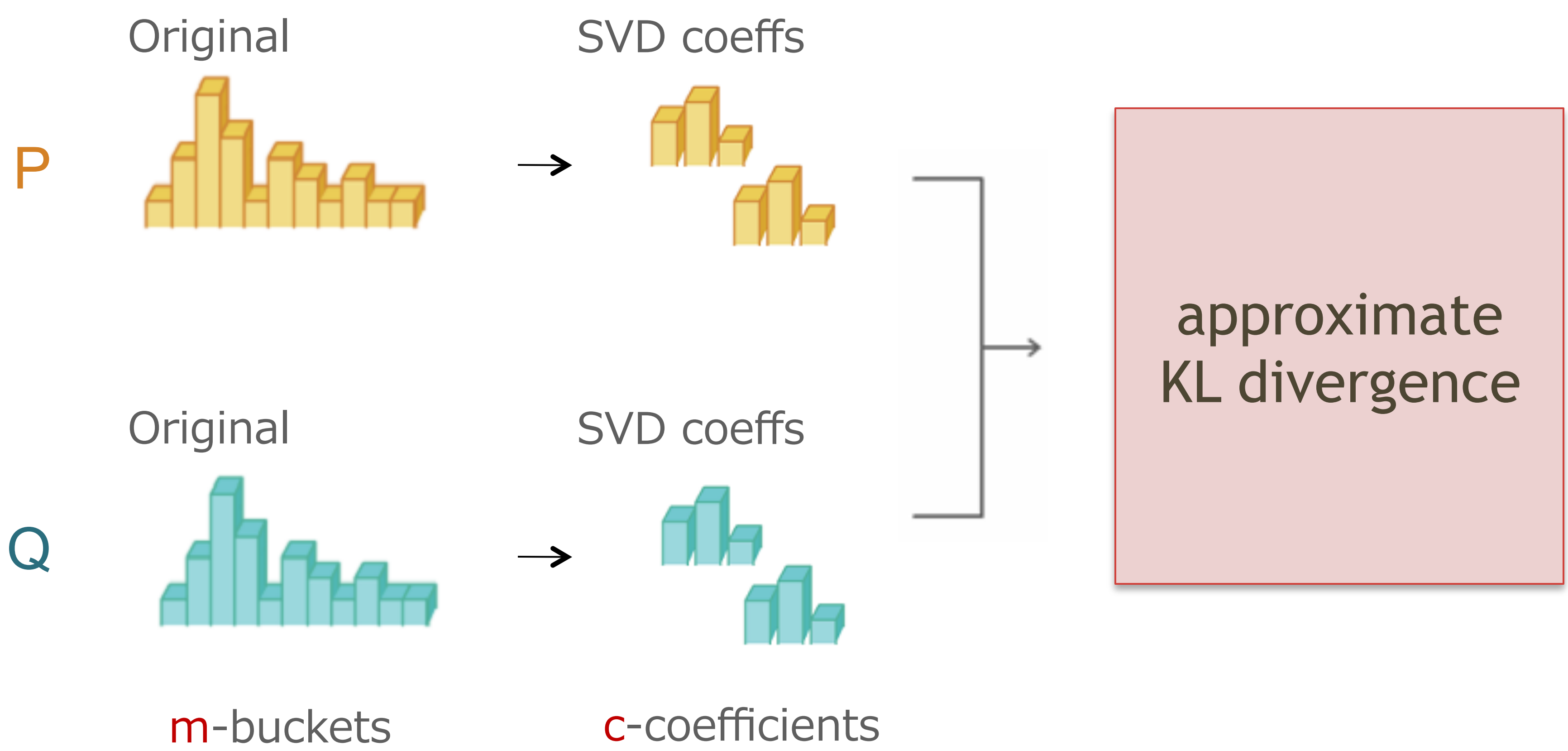
sp_i : The SVD of p_i

$s\hat{p}_i$: The SVD of $\log(p_i)$

Dimension : m c

Enhanced D-Search

- Approximate KL divergence
  - Compute the KL divergence from the SVDs



Enhanced D-Search

m: # of buckets of a histogram
c: # of SVD coefficients
(m >> c)

Theorem 1

Let

sp_i and sq_i be the SVD of p_i and q_i resp.

$s\hat{p}_i$ and $\hat{s}q_i$ be the SVD of $\log p_i$ and $\log q_i$ resp.

We have

$$d_{SKL}(P,Q) = \sum_{i=1}^{m} (p_i - q_i) \cdot \log\left(\frac{p_i}{q_i}\right)$$

$$= \sum_{i=1}^{m} (p_i - q_i) \cdot (\log p_i - \log q_i)$$

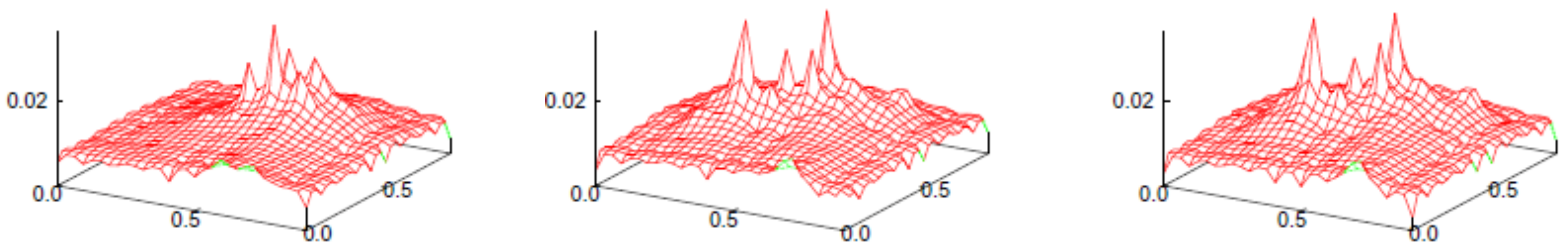
$$\approx \frac{1}{2} \cdot \sum_{i=1}^{c} \begin{pmatrix} (sp_i - \hat{s}q_i)^2 + (sq_i - \hat{s}p_i)^2 \\ -(sp_i - \hat{s}p_i)^2 - (sq_i - \hat{s}q_i)^2 \end{pmatrix}$$

Approx. KL divergence can be computed from SVD coefficients

Enhanced D-Search

- Multi-Step Sequential Scan
  - SVD-based approx. of distribution from MoCap
  - Represented by a 10*10 bucketized histogram
  - (Full coefficients c = m = 100)

(a) c = 1 (b) c = 16 (c) Original data



Enhanced D-Search

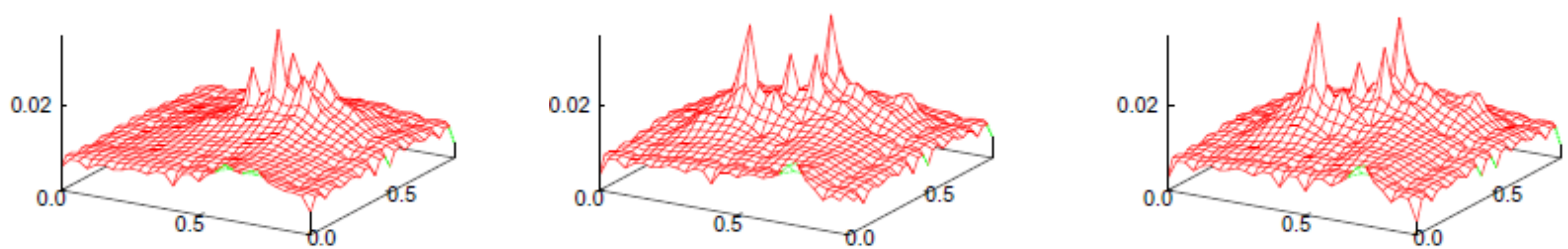
- <u>Gradual refinement of the approximation:</u>

  For the first step,

  - Compute the approx. distance from the coarsest version of the distribution ((a) c=1)
  - If the distance is greater than D_{cb} , we can prune it

Step 1

(a) c = 1 (b) c = 16 (c) Original data



Enhanced D-Search

- <u>Gradual refinement of the approximation:</u>

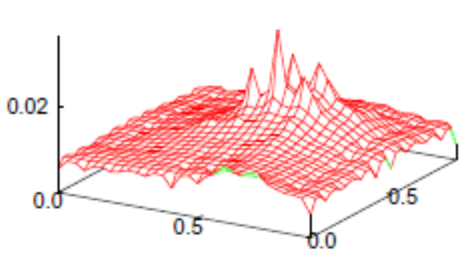
  Otherwise,

  for the second step,

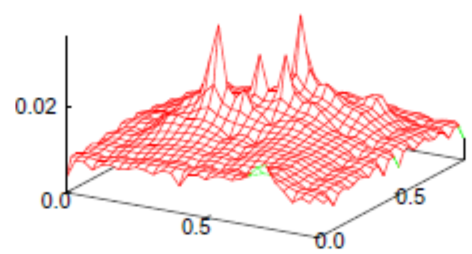
  - Compute the approx. distance from the more accurate version of the distribution ((b) c=16)

Step 2

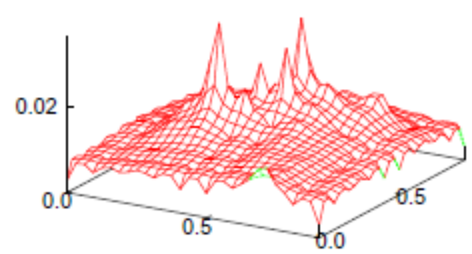
(a) c = 1



(b) c = 16



(c) Original data



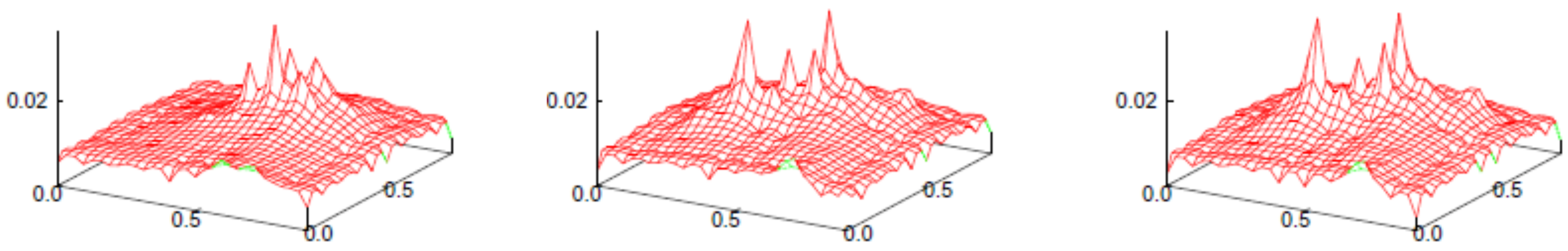
Enhanced D-Search

- <u>Gradual refinement of the approximation:</u>

  If the approx. distance does not exceed D_{cb}
  for the third step,
  - Compute the exact distance
    from the original distribution ((c) original data)

Step 3

(a) c = 1 (b) c = 16 (c) Original data



Enhanced D-Search

- Gradual refinement of the approximation:

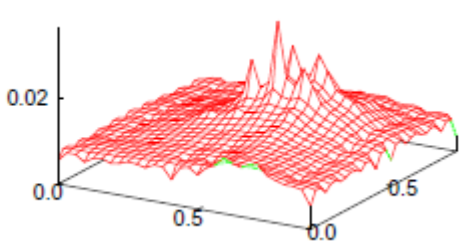
  For the final step,

  If the exact distance does not exceed D_{cb}

  - Update the answer candidate and D_{cb}

- Repeat this procedure for every distribution

(a) c = 1 (b) c = 16 (c) Original data



Time complexity

- Computation for KL divergence

Naïve method	D-Search
$O(mn)$	$O(n)$

n: # of input distributions
m: # of buckets of histogram
c : # of SVD coefficients we use

D-Search :

- requires O(cn)

• c is a small constant and negligible

Space complexity

- Space for our method

Naïve method	D-Search
$O(mn)$	$O(m+n)$

D-Search :

- allocates space to store histogram of m buckets
- allocates O(cn) space for computing the criterion
- We obtain O(m + cn)

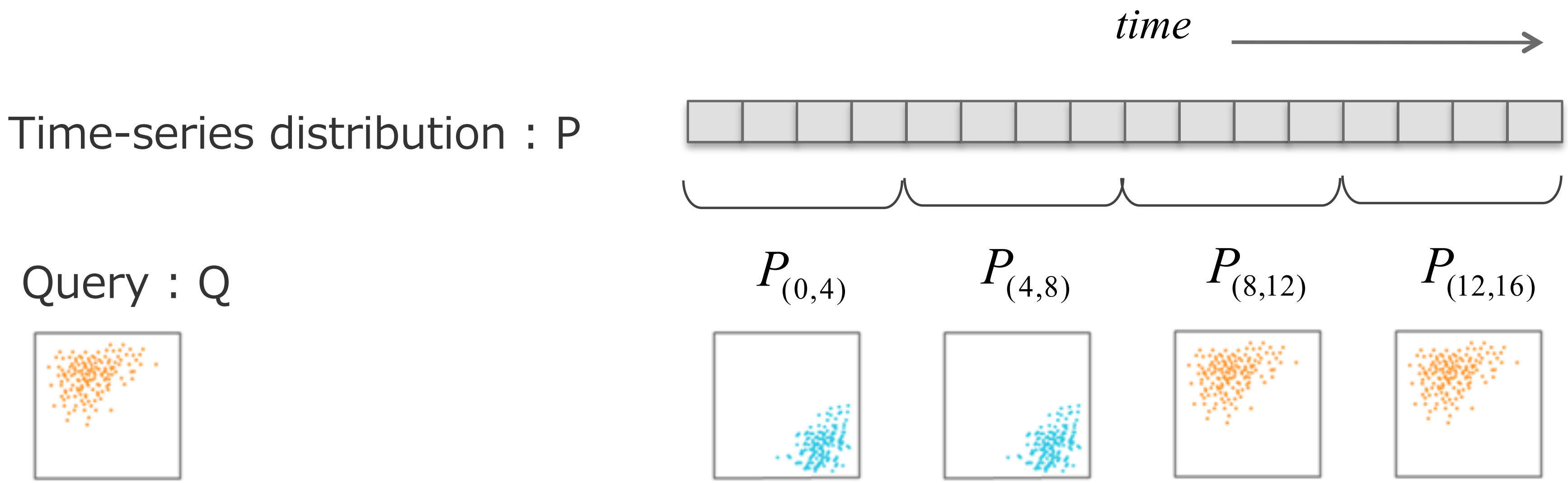
* c is a small constant and negligible

Outline

- Introduction
- Background
- D-Search
- **Time-series distribution mining**
- Experiments
- Conclusions

Time-series distribution mining

- Problem:
  - Given time-series distribution P and query Q,
  - Finds similar subsequences



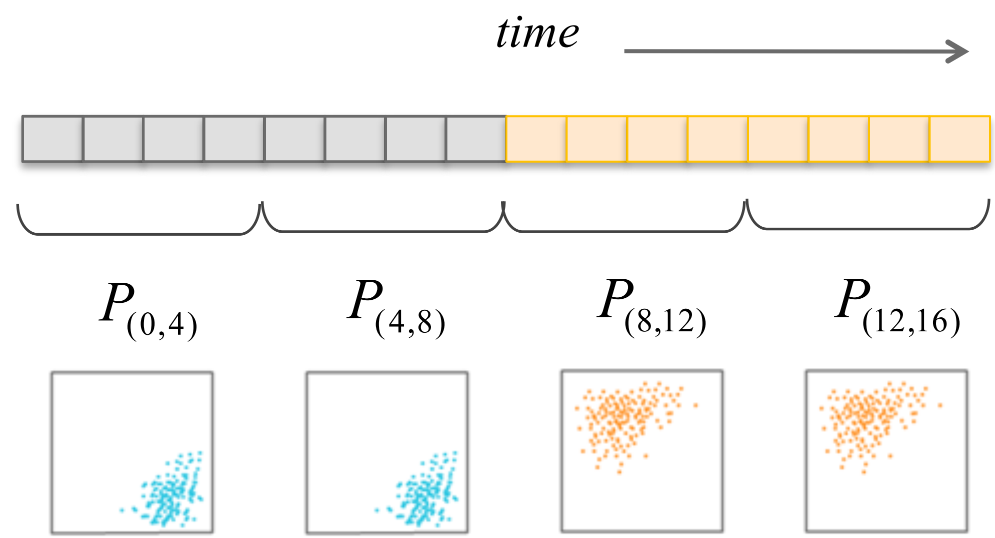
Time-series distribution mining

Problem:

Example:

We want to find three subsequences : 8-12sec., 12-16sec., 8-16sec.

Time-series distribution : P



Query : Q

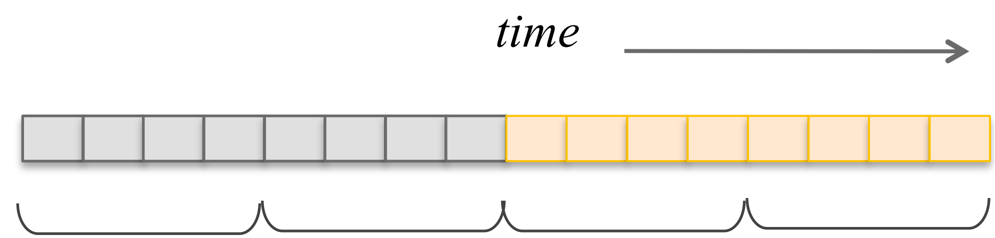
Time-series distribution mining

Problem:

Example:

We want to find three subsequences : 8-12sec., 12-16sec., 8-16sec.

Time-series distribution : P

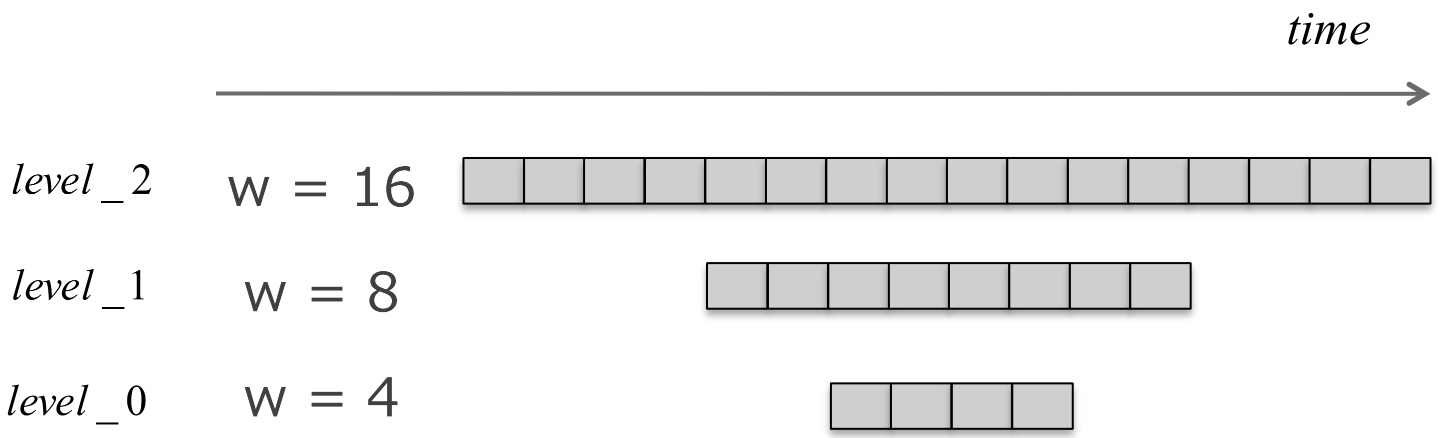


Q: How do we efficiently find the similar subsequences for multiple lengths?

Time-series distribution mining

A: Use hierarchical window sizes

Main idea: Geometric progression of windows sizes



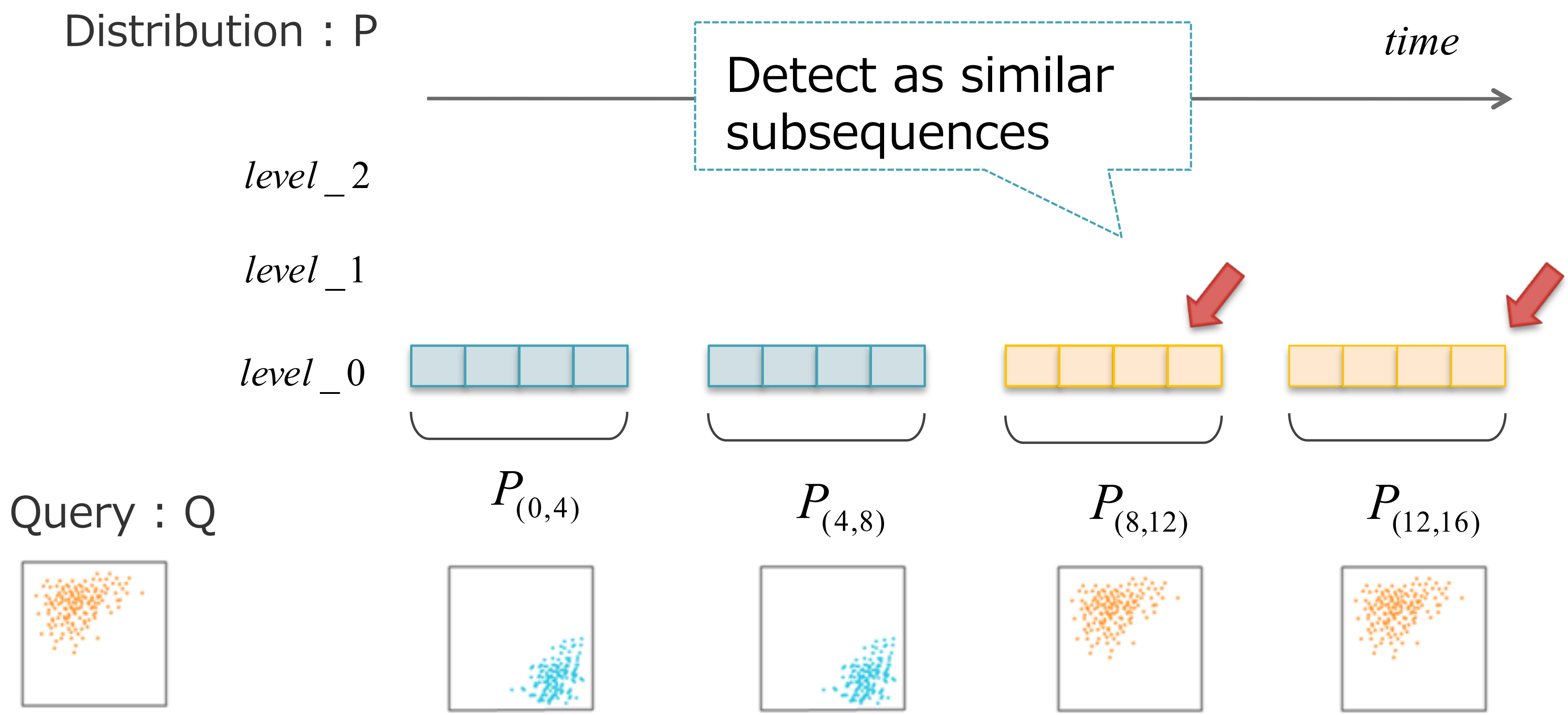
Query : Q

$$w = w_0 \cdot 2^l \qquad l = \{0,1,2,3,\dots\}$$

- The size of the window set can be reduced

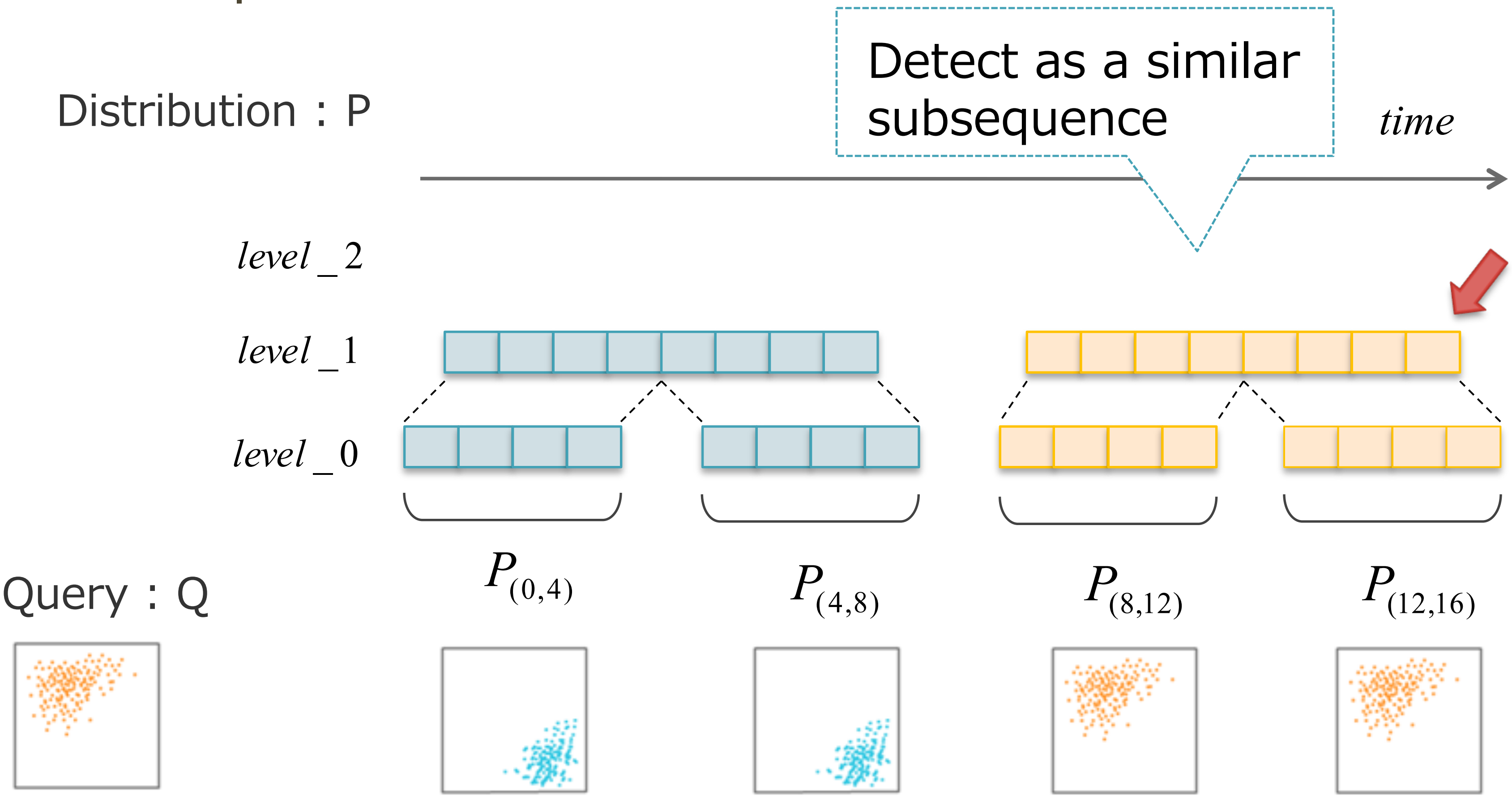
Time-series distribution mining

- How to detect similar subsequences
  - Example: at the level 0



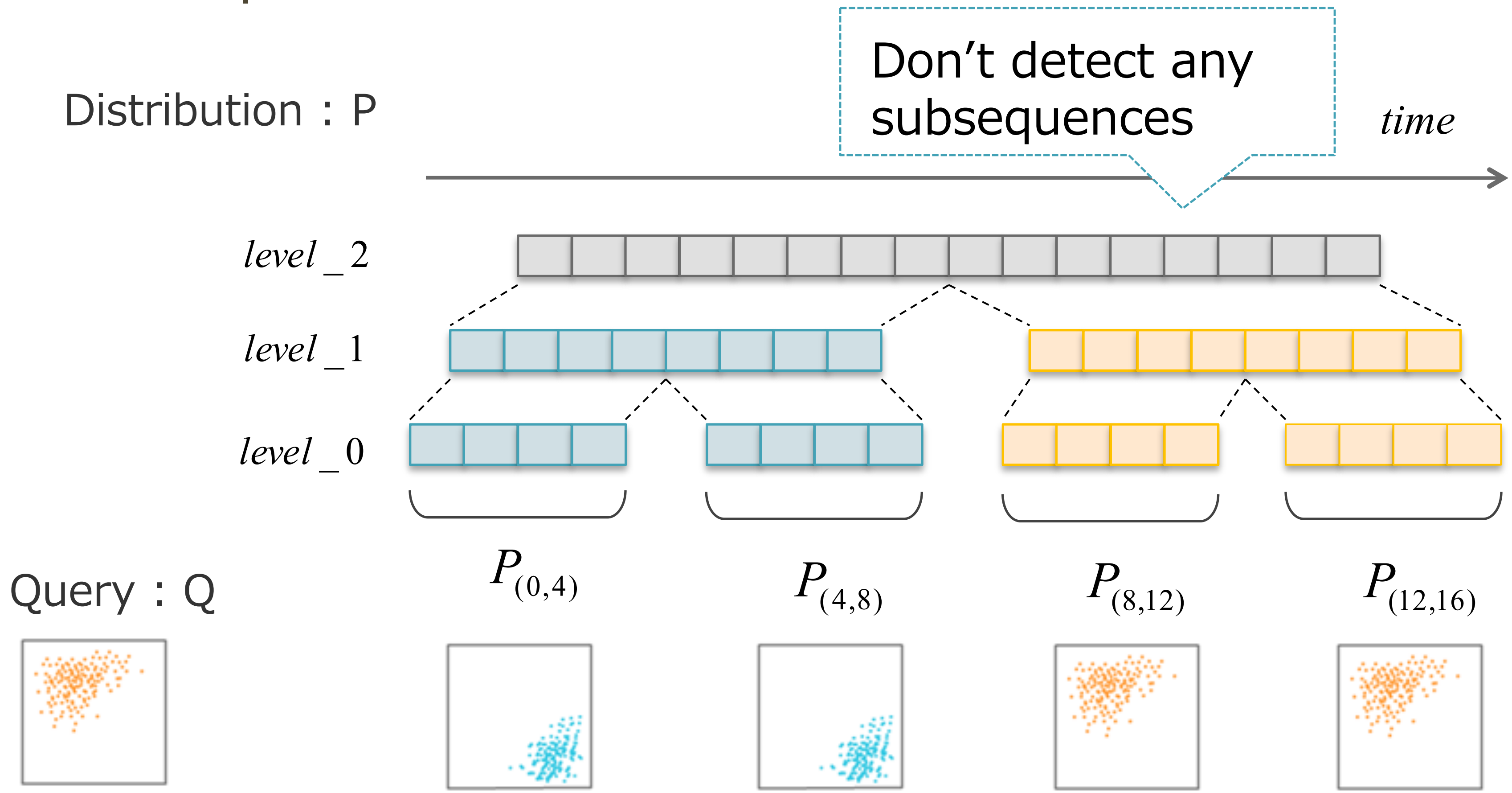
Time-series distribution mining

- How to detect similar subsequences
  - Example: at the level 1



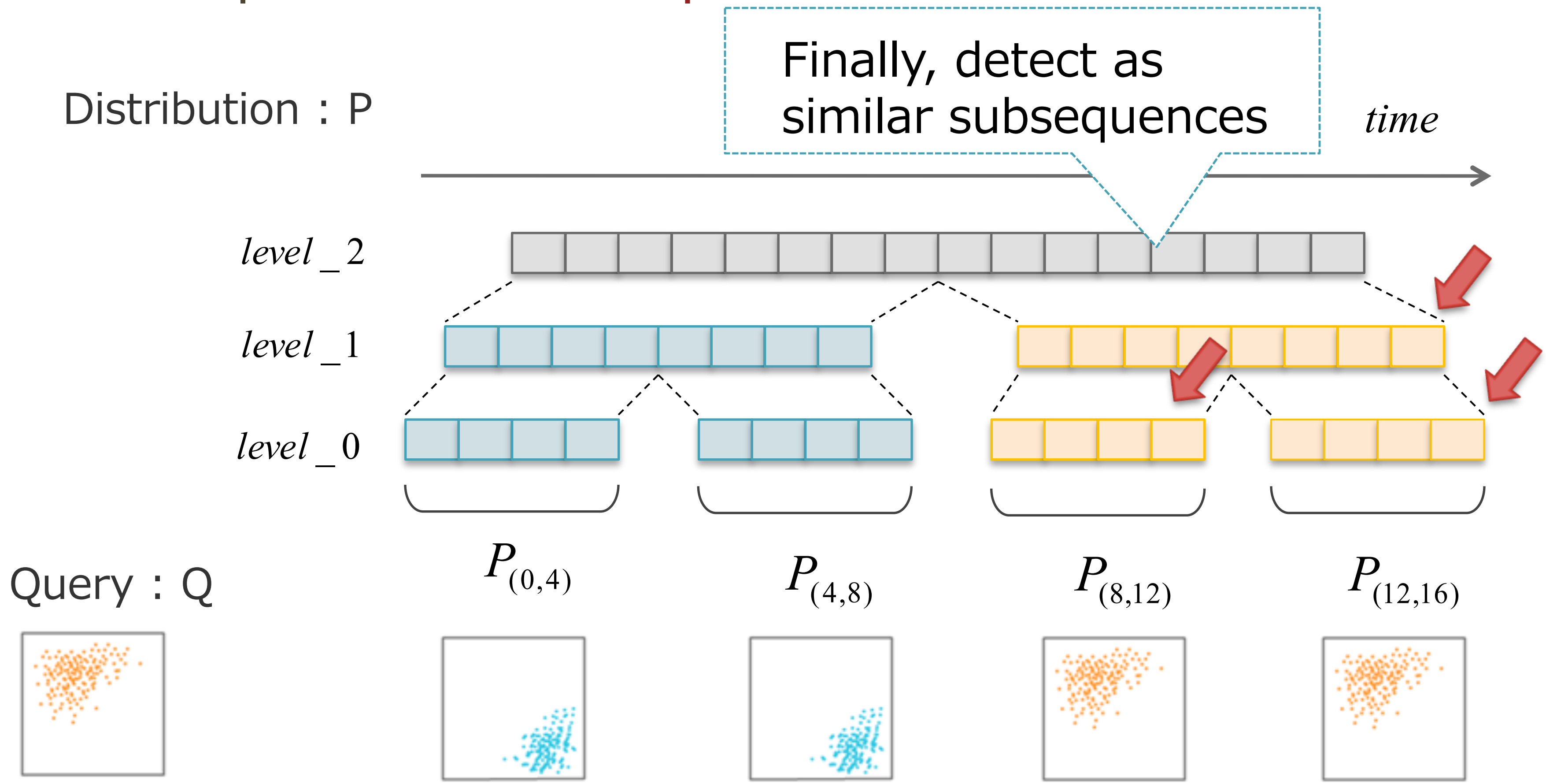
Time-series distribution mining

- How to detect similar subsequences
  - Example: at the level 2



Time-series distribution mining

- How to detect similar subsequences
  - Example: at the multiple levels



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Experimental evaluation

- The experiments were designed to answer the three questions:

1. Effectiveness

How successful is **D-Search (enhanced)** in capturing time-series distribution patterns?

2. Speed

How does **D-Search** scale with the sequence lengths n in terms of the computational time?

3. Quality

How well does **D-Search** approximate the KL divergence?

Experimental evaluation

- We carried out experiments on real datasets:

Numerical data

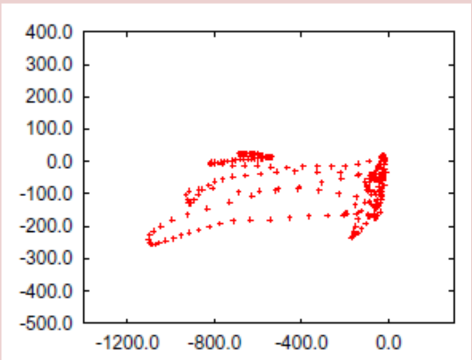
- **Motion capture**
  - It contains 26 sequences, each of which is a series of simple motions such as walking, running, jumping
- **EEG**
  - It is from a large study that examined the EEG correlates of alcoholism. There were two subject groups: alcoholic and control

Categorical data

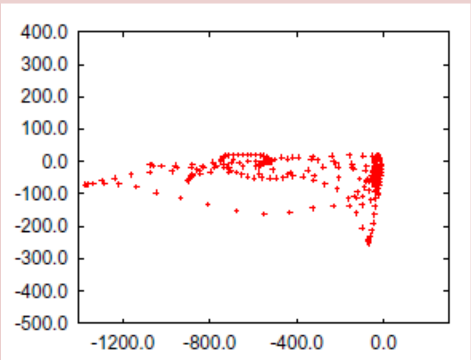
- **On-demand TV**
  - Dataset from the on-demand TV service. It contains a list of content ID, Date, user ID
- **Music store**
  - This dataset consists of the purchasing records from an on-line music store obtained over 16 months

Case studies

(1) Motion capture

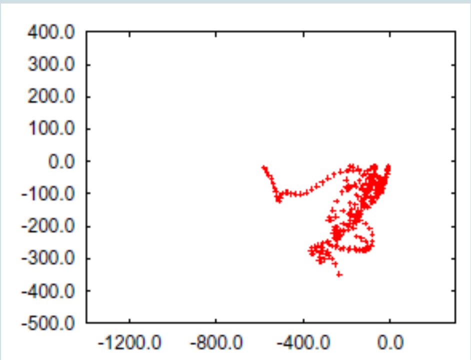


Query : jumping



#1 : jumping

ID:14_14 (2-4sec.)



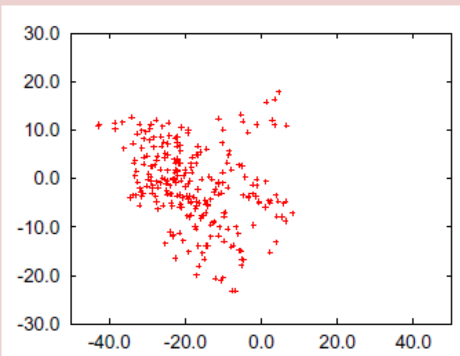
#2 : jogging

ID:14_14 (4-6sec.)

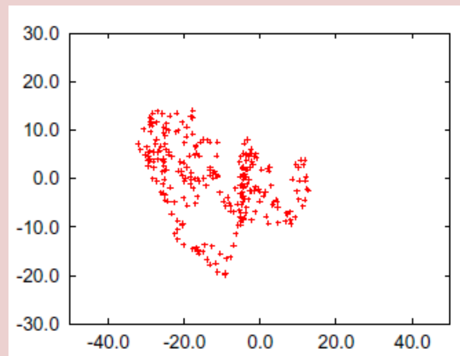
- D-Search can identify similar subsequences
  - Query and #1 both correspond to a **jumping** motion
  - #2 corresponds to a **jogging** motion

Case studies

(2) EEG (Alcohol or Control)

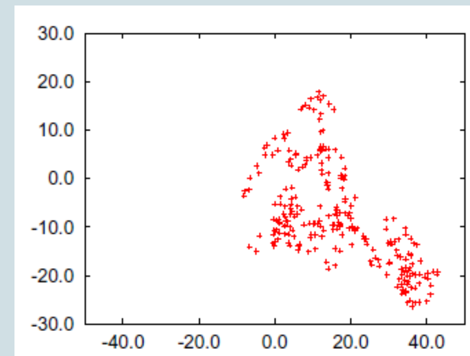


Query : alcohol



#1 : alcohol

co3a (55-56sec.)



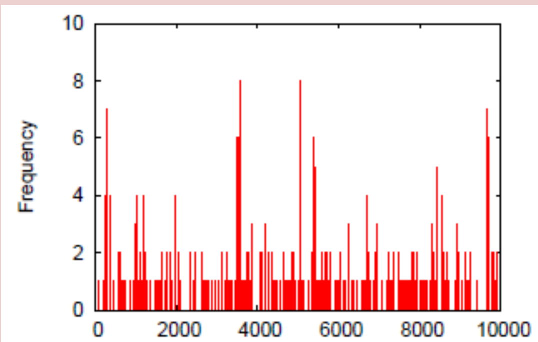
#2 : control

co2c (74-75sec.)

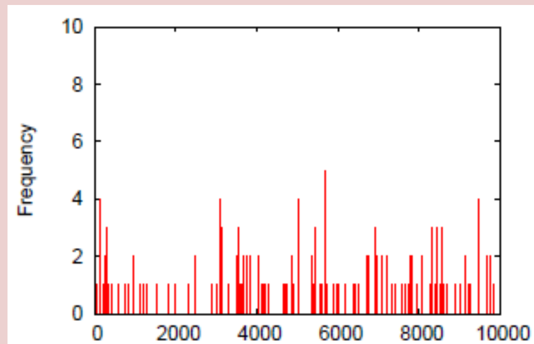
- Our approach is also useful for classification
  - Query and #1 are classified into the same group
  - #2 goes to another group (it belongs to “control”)

Case studies

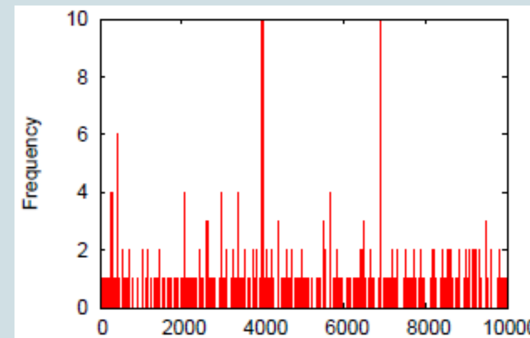
(3) On-demand TV (distribution of users)



Query : tennis final



#1 : tennis semifinal

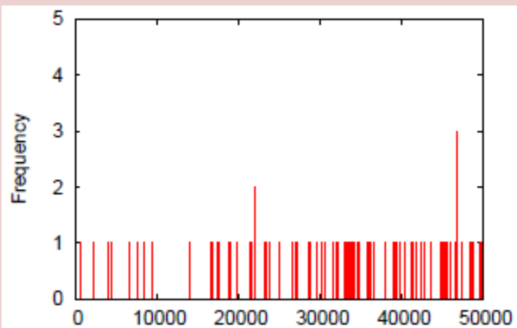


#2 : cooking

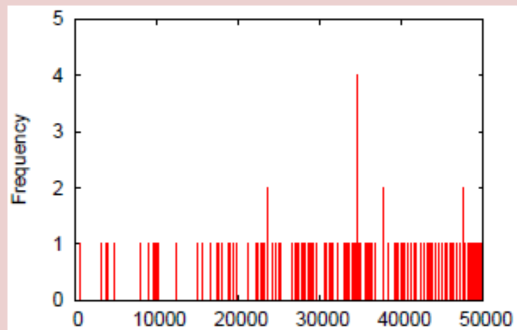
- D-Search can find similar trends in Q and #1
  - “Australian Open Tennis final” and “Australian Open Tennis semi-final”

Case studies

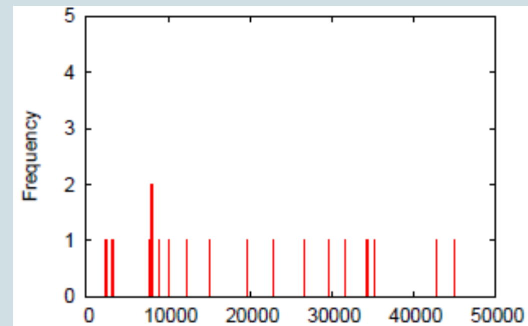
(4) Music Store (distribution of purchasers)



Query : John Lennon



#1 : John Lennon



#2 : Nat King Cole

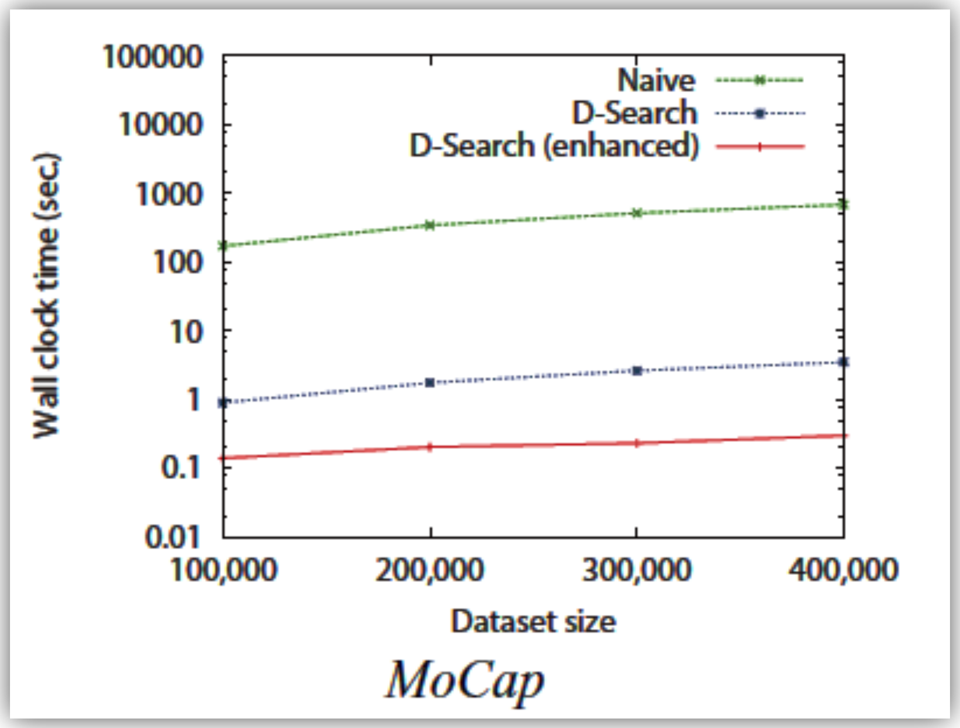
- D-Search can find similar purchasers groups
  - the songs of the same artist are identified as the same purchasers' group

Computation cost

- Compared with the naïve approach:
  - **Naïve**
    use all histogram buckets
  - **D-Search (basic)**
    use only selected buckets (largest values),
    and use the multi-step sequential scan
  - **D-Search (enhanced)**
    use the SVD coefficients,
    and use the multi-step sequential scan

Computation cost

- Compared with the naïve approach:

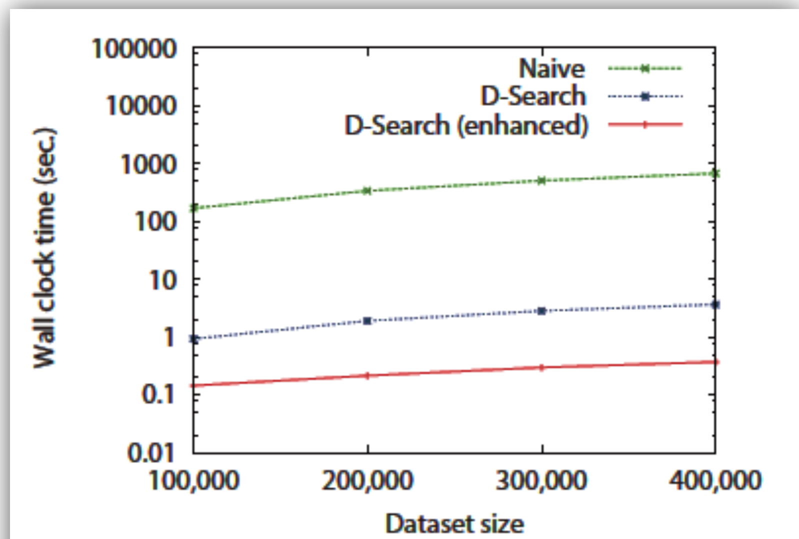


D-Search provides a reduction in computation time

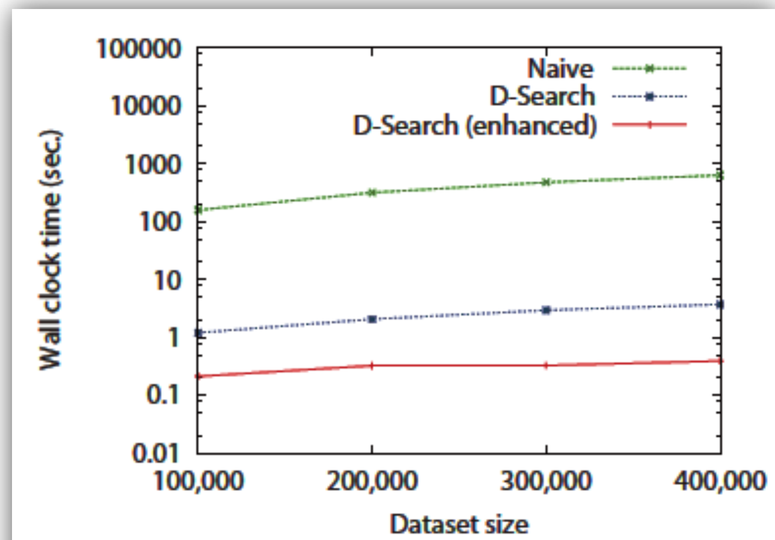
(up to 2,300 times faster than **Naïve**, 10 times faster than **D-Search (basic)**)

Computation cost

- Compared with the naïve approach:



EEG



Ondemand TV

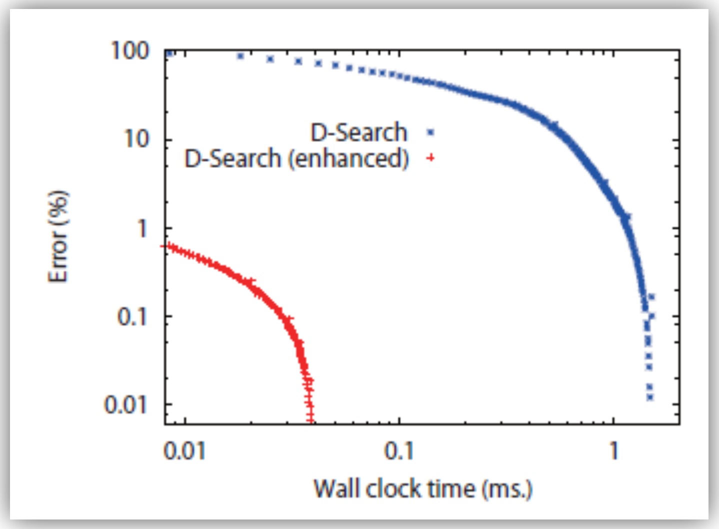
D-Search provides a reduction in computation time

(up to 2,300 times faster than **Naïve**, 10 times faster than **D-Search (basic)**)

Quality of proposed methods

Approximation Quality of SVD

- Trade-off between quality and cost



Ex. On demand TV

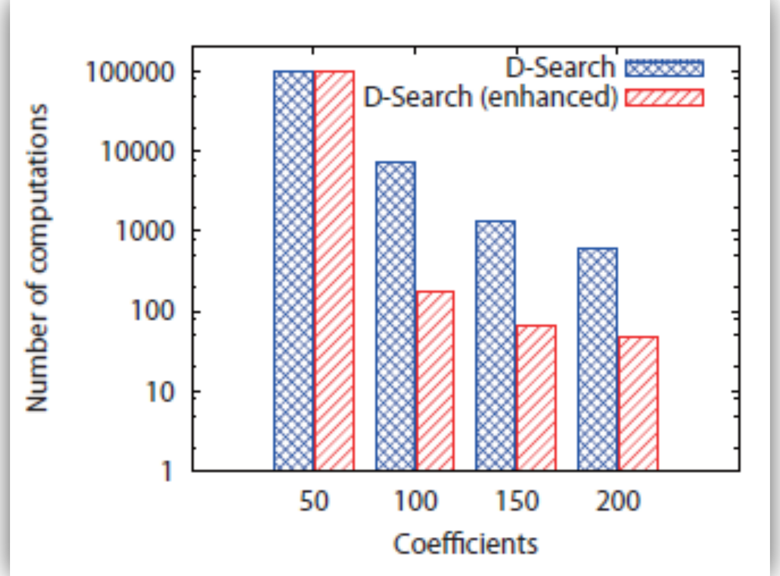
- Scatter plot of computation cost vs. approx. quality
- vary the number of c for each approx. technique

- SVD gives significantly lower approximation error, for the same computation time

Quality of proposed methods

Effect of the multi-step sequential scan

- How often each approximation was used?



Ex. On demand TV

D-Search efficiently prunes a large number of candidates, which leads to a significant reduction in the search cost

Outline

- Introduction
- Background
- D-Search
- Time-series distribution mining
- Experiments
- Conclusions

Conclusions

- Addressed the problem of distribution search
- Proposed a fast and effective method to solve it
  - Lower bounding KL divergence
  - Multi-step sequential scan
  - SVD-based approximate KL divergence
- Extended to time-series distribution mining
- Experiments show that our approach is faster than naïve implementation (up to 2,300 times)

Scalable Algorithms for Distribution Search

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